

Examples

# MathML in ConTEXT

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PRAGMA ADE

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This document shows a few formulas coded in MathML and typeset by ConT<sub>E</sub>Xt. The examples are taken from an old copy of ‘Handbook of Chemistry and Physics’ as well as ‘Wiskunde voor het HBO (R. van Asselt et al.)’. We assume no responsibility for the coding being 100% all correct.

These examples are typeset using the default settings. There are several ways to influence the look and feel of a formula. Details on how to process MathML can be found in the xml related documentation that comes with ConT<sub>E</sub>Xt.

You can get more information on ConT<sub>E</sub>Xt at our website, in T<sub>E</sub>X usergroup publications and in (the archives of) the ConT<sub>E</sub>Xt mailing list.

Hans Hagen

Hasselt, January 2001 / June 2008 / June 2015

[www.pragma-ade.com](http://www.pragma-ade.com)

# Derivatives

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$$\frac{da}{dx} = 0$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> a </ci>
    </apply>
    <ci> 0 </ci>
  </apply>
</math>
```

$$\frac{dx}{dx} = 1$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> x </ci>
    </apply>
    <cn> 1 </cn>
  </apply>
</math>
```

$$\frac{d(au)}{dx} = a \frac{du}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <times/>
        <ci> a </ci>
        <ci> u </ci>
      </apply>
    </apply>
    <apply> <times/>
      <ci> a </ci>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> u </ci>
      </apply>
    </apply>
  </math>
```

$$\frac{d(u + v + w)}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <plus/>
        <ci> u </ci>
        <ci> v </ci>
        <ci> w </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> u </ci>
      </apply>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> v </ci>
      </apply>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> w </ci>
      </apply>
    </apply>
  </apply>
</math>
```



$$\frac{d(uv)}{dx} = u \frac{du}{dx} + v \frac{dv}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply <eq/>
    <apply <diff/>
      <bvar <ci> x </ci> </bvar>
      <apply <times/>
        <ci> u </ci>
        <ci> v </ci>
      </apply>
    </apply>
    <apply <plus/>
      <apply <times/>
        <ci> u </ci>
        <apply <diff/>
          <bvar <ci> x </ci> </bvar>
          <ci> u </ci>
        </apply>
      </apply>
      <apply <times/>
        <ci> v </ci>
        <apply <diff/>
          <bvar <ci> x </ci> </bvar>
          <ci> v </ci>
        </apply>
      </apply>
    </apply>
  </math>
```

$$\frac{d(uvw)}{dx} = vw \frac{du}{dx} + uw \frac{dv}{dx} + uv \frac{dw}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <times/>
        <ci> u </ci>
        <ci> v </ci>
        <ci> w </ci>
      </apply>
    </apply>
    <apply> <plus/>
      <apply> <times/>
        <ci> v </ci>
        <ci> w </ci>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> u </ci>
      </apply>
    </apply>
    <apply> <times/>
      <ci> u </ci>
      <ci> w </ci>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> v </ci>
    </apply>
  </apply>
  <apply> <times/>
    <ci> u </ci>
    <ci> v </ci>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> w </ci>
  </apply>
</apply>
</math>
```

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{1}{v}\frac{du}{dx} - \frac{u}{v^2}\frac{dv}{dx}$$

```

<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <ci> u </ci>
        <ci> v </ci>
      </apply>
    </apply>
    <apply> <divide/>
      <apply> <minus/>
        <apply> <times/>
          <ci> v </ci>
          <apply> <diff/>
            <bvar> <ci> x </ci> </bvar>
            <ci> u </ci>
          </apply>
        </apply>
        <apply> <times/>
          <ci> u </ci>
          <apply> <diff/>
            <bvar> <ci> x </ci> </bvar>
            <ci> v </ci>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <power/>
      <ci> v </ci>
      <cn> 2 </cn>
    </apply>
  </apply>
  <apply> <minus/>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <ci> v </ci>
      </apply>
      <apply> <diff/>
        <bvar> <ci> x </ci> </bvar>
        <ci> u </ci>
      </apply>
    </apply>
  </apply>
  <apply> <times/>
    <apply> <divide/>
      <cn> u </cn>
      <apply> <power/>
        <ci> v </ci>
        <cn> 2 </cn>
      </apply>
    </apply>
  </apply>
  <apply> <diff/>
    <bvar> <ci> x </ci> </bvar>
    <ci> v </ci>
  </apply>
</math>

```

$$\frac{d(u^n)}{dx} = n(u) \frac{du}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <power/>
        <ci> u </ci>
        <ci> n </ci>
      </apply>
    </apply>
    <apply> <times/>
      <ci> n </ci>
      <apply> <power/>
        <ci> u </ci>
        <apply> <minus/>
          <ci> n </ci>
          <cn> 1 </cn>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> u </ci>
    </apply>
  </apply>
</math>
```

$$\frac{d\sqrt{u}}{dx} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <root/>
        <ci> u </ci>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <cn> 2 </cn>
          <apply> <root/>
            <ci> u </ci>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> u </ci>
    </apply>
  </apply>
</math>
```

$$\frac{d\left(\frac{1}{u}\right)}{dx} = -\frac{1}{u^2} \frac{du}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <ci> u </ci>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <minus/>
        <apply> <divide/>
          <cn> 1 </cn>
          <apply> <power/>
            <ci> u </ci>
            <cn> 2 </cn>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> u </ci>
    </apply>
  </apply>
</math>
```

$$\frac{d\left(\frac{1}{u^n}\right)}{dx} = -\frac{n}{u} \frac{du}{dx}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <power/>
          <ci> u </ci>
          <cn> n </cn>
        </apply>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <minus/>
        <apply> <divide/>
          <ci> n </ci>
          <apply> <power/>
            <ci> u </ci>
            <apply> <plus/>
              <ci> n </ci>
              <cn> 1 </cn>
            </apply>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <ci> u </ci>
    </apply>
  </apply>
</math>
```

$$\frac{d}{dx} = \frac{d \log(u + \sqrt{u^2 + 1})}{dx} = \frac{1}{\sqrt{u^2 + 1}} \frac{du}{dx}$$

```

<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <inverse/>
        <apply> <sinh/>
          <ci> u </ci>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <log/>
        <apply> <plus/>
          <ci> u </ci>
          <apply> <root/>
            <apply> <plus/>
              <apply> <power/>
                <ci> u </ci>
                <cn> 2 </cn>
              </apply>
            <cn> 1 </cn>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <times/>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <root/>
          <apply> <plus/>
            <apply> <power/>
              <ci> u </ci>
              <cn> 2 </cn>
            </apply>
          <cn> 1 </cn>
        </apply>
      </apply>
    </apply>
    <apply> <diff/>
      <bvar> <ci> x </ci> </bvar>

```

```

  <ci> u </ci>
  </apply>
</math>

```



$$\frac{d \left( \int_p^q f(x, a) dx \right)}{da}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <diff/>
      <bvar> <ci> a </ci> </bvar>
      <apply> <int/>
        <lowlimit> <ci> p </ci> </lowlimit>
        <uplimit> <ci> q </ci> </uplimit>
        <bvar> <ci> x </ci> </bvar>
        <apply>
          <fn> <ci> f </ci> </fn>
          <ci> x </ci>
          <ci> a </ci>
        </apply>
      </apply>
    </apply>
  </math>
```

# Integrals

[pc-i-022](#)

[pc-i-380](#)

$$\int \left( \frac{1}{x\sqrt{a^2 \pm x^2}} \right) dx = -\frac{1}{a} \log \frac{a + \sqrt{a^2 \pm x^2}}{x}$$

```

<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <int/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <ci> x </ci>
          <apply> <root/>
            <apply> <fn> <ci> &plusminus; </ci> </fn>
            <apply> <power/>
              <ci> a </ci>
              <cn> 2 </cn>
            </apply>
            <apply> <power/>
              <ci> x </ci>
              <cn> 2 </cn>
            </apply>
          </apply>
        </apply>
      </apply>
    </apply>
    <apply> <minus/>
      <apply> <times/>
        <apply> <divide/>
          <cn> 1 </cn> <ci> a </ci>
        </apply>
        <apply> <log/>
          <apply> <divide/>
            <apply> <plus/>
              <ci> a </ci>
              <apply> <root/>
                <apply> <fn> <ci> &plusminus; </ci> </fn>
                <apply> <power/>
                  <ci> a </ci>
                  <cn> 2 </cn>
                </apply>
              </apply>
            <apply> <power/>
              <ci> x </ci>
              <cn> 2 </cn>
            </apply>
          </apply>
        </apply>
      </apply>
    </apply>
  </math>

```

$$\int \left( \frac{1}{\cos(ax)(1 \pm \sin(ax))} \right) dx = \left( \frac{1}{2a(1 \pm \sin(ax))} \right) + \frac{1}{2a} \log \tan \left( \frac{\pi}{4} + \frac{ax}{2} \right)$$

```

<math xmlns='http://www.w3c.org/mathml' version='2.0'
  <apply> <eq/>
    <apply> <int/>
      <bvar> <ci> x </ci> </bvar>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <apply> <cos/>
            <apply> <times/>
              <ci> a </ci>
              <ci> x </ci>
            </apply>
          </apply>
        </apply>
      <apply> <fn> <ci> &plusminus; </ci> </fn>
        <cn> 1 </cn>
        <apply> <sin/>
          <apply> <times/>
            <ci> a </ci>
            <ci> x </ci>
          </apply>
        </apply>
      </apply>
    </apply>
  <apply> <plus/>
    <apply> <fn> <ci> &minusplus; </ci> </fn>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <times/>
          <cn> 2 </cn>
          <ci> a </ci>
          <apply> <fn> <ci> &plusminus; </ci> </fn>
            <cn> 1 </cn>
            <apply> <sin/>
              <apply> <times/>
                <ci> a </ci>
                <ci> x </ci>
              </apply>
            </apply>
          </apply>
        </apply>
      </apply>
    </apply>
  </math>

```

# Series

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+

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <minus/>
        <apply> <divide/>
          <cn> 1 </cn>
          <cn> 3 </cn>
        </apply>
      </apply>
    <apply> <divide/>
      <cn> 1 </cn>
      <cn> 5 </cn>
    </apply>
    <apply> <minus/>
      <apply> <divide/>
        <cn> 1 </cn>
        <cn> 7 </cn>
      </apply>
    </apply>
    <ci> &cdots; </ci>
  </apply>
  <apply> <divide/>
    <ci> &pi; </ci>
    <cn> 4 </cn>
  </apply>
</apply>
</math>
```

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <power/>
          <cn> 2 </cn>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
    <apply> <divide/>
      <cn> 1 </cn>
      <apply> <power/>
        <cn> 3 </cn>
        <cn> 2 </cn>
      </apply>
    </apply>
    <apply> <divide/>
      <cn> 1 </cn>
      <apply> <power/>
        <cn> 4 </cn>
        <cn> 2 </cn>
      </apply>
    </apply>
    <ci> &cdots; </ci>
  </apply>
  <apply> <divide/>
    <apply> <power/>
      <ci> &pi; </ci>
      <cn> 2 </cn>
    </apply>
    <cn> 6 </cn>
  </apply>
</math>
```

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <eq/>
    <apply> <plus/>
      <cn> 1 </cn>
      <apply> <minus/>
        <apply> <divide/>
          <cn> 1 </cn>
          <apply> <power/>
            <cn> 2 </cn>
            <cn> 2 </cn>
          </apply>
        </apply>
      </apply>
    <apply> <divide/>
      <cn> 1 </cn>
      <apply> <power/>
        <cn> 3 </cn>
        <cn> 2 </cn>
      </apply>
    </apply>
    <apply> <minus/>
      <apply> <divide/>
        <cn> 1 </cn>
        <apply> <power/>
          <cn> 4 </cn>
          <cn> 2 </cn>
        </apply>
      </apply>
    </apply>
    <ci> &cdots; </ci>
  </apply>
  <apply> <divide/>
    <apply> <power/>
      <ci> &pi; </ci>
      <cn> 2 </cn>
    </apply>
    <cn> 12 </cn>
  </apply>
</math>
```



$$\forall x \in \mathbb{R} \quad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

```

<math xmlns='http://www.w3c.org/mathml' version='2.0'>
  <apply> <forall/>
    <condition>
      <apply> <in/>
        <ci> x </ci>
        <ci> &reals; </ci>
      </apply>
    </condition>
  <apply> <eq/>
    <apply> <power/>
      <ci> &exponentiale; </ci>
      <ci> x </ci>
    </apply>
    <apply> <plus/>
      <cn> 1 </cn>
      <ci> x </ci>
      <apply> <divide/>
        <apply> <power/>
          <ci> x </ci>
          <cn> 2 </cn>
        </apply>
        <apply> <factorial/>
          <cn> 2 </cn>
        </apply>
      </apply>
      <apply> <divide/>
        <apply> <power/>
          <ci> x </ci>
          <cn> 3 </cn>
        </apply>
        <apply> <factorial/>
          <cn> 3 </cn>
        </apply>
      </apply>
      <ci> &cdots; </ci>
    </apply>
    <apply> <divide/>
      <apply> <power/>
        <ci> x </ci>
        <ci> n </ci>
      </apply>
      <apply> <factorial/>
        <ci> n </ci>
      </apply>
    </apply>
  </apply>
</math>

```

$$\forall x \in \mathbb{R} \quad (e) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^n}{n!} \dots$$

```

<math xmlns='http://www.w3c.org/mathml' version='2.0'
  <apply> <forall/>
    <condition>
      <apply> <in/>
        <ci> x </ci>
        <ci> &reals; </ci>
      </apply>
    </condition>
    <apply> <eq/>
      <apply> <power/>
        <ci> &exponentiale; </ci>
        <apply> <minus/>
          <ci> x </ci>
        </apply>
      </apply>
      <apply> <plus/>
        <cn> 1 </cn>
        <apply> <minus/>
          <ci> x </ci>
        </apply>
      </apply>
      <apply> <divide/>
        <apply> <power/>
          <ci> x </ci>
          <cn> 2 </cn>
        </apply>
        <apply> <factorial/>
          <cn> 2 </cn>
        </apply>
      </apply>
      <apply> <minus/>
        <apply> <divide/>
          <apply> <power/>
            <ci> x </ci>
            <cn> 3 </cn>
          </apply>
          <apply> <factorial/>
            <cn> 3 </cn>
          </apply>
        </apply>
      </apply>
      <ci> &cdots; </ci>
      <apply> <times/>
        <apply> <power/>
          <apply> <minus/>
            <cn> 1 </cn>
          </apply>
          <ci> n </ci>
        </apply>
        <apply> <divide/>
          <apply> <power/>
            <ci> x </ci>
            <ci> n </ci>
          </apply>
          <apply> <factorial/>
            <ci> n </ci>
          </apply>
        </apply>
        <ci> &cdots; </ci>
      </apply>
    </math>

```

# Logs

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$$\forall a > 0 \wedge b > 0 \mid \log_g ab = \log_g a + \log_g b$$

```
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$$\forall a > 0 \wedge b > 0 \mid \log_g \frac{a}{b} = \log_g a - \log_g b$$

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$$\forall b \in \mathbb{R} \wedge a > 0 \mid \log_g a^b = b \log_g a$$

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```

$$\forall a > 0 \quad \left| \log_g a = \frac{\log_p a}{\log_p g} \right.$$

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# Goniometrics

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$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

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$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

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$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

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$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

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$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

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$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

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$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$$

```
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            <ci> q </ci>
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    </apply>
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```

$$\sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2}$$

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```



$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$$

```
<math xmlns='http://www.w3c.org/mathml' version='2.0'>
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            <ci> q </ci>
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          <cn> 2 </cn>
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```

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}$$

```
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```

$$2 \sin \alpha \cos \beta = \sin (\alpha + \beta) + \sin (\alpha - \beta)$$

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          <ci> &beta; </ci>
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```

$$2 \cos \alpha \sin \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

```
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          <ci> &beta; </ci>
        </apply>
      </apply>
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```

$$2 \cos \alpha \cos \beta = \cos (\alpha + \beta) + \cos (\alpha - \beta)$$

```
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$$-2 \sin \alpha \cos \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

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          <ci> &beta; </ci>
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```

$$\forall \triangle ABC \left| \frac{a}{\sin \alpha} + \frac{b}{\sin \beta} + \frac{c}{\sin \gamma} \right.$$

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```

$$\forall \triangle ABC \begin{cases} a^2 = b^2 + c^2 - 2bc \cos \alpha \\ b^2 = a^2 + c^2 - 2ac \cos \beta \\ c^2 = a^2 + b^2 - 2ab \cos \gamma \end{cases}$$

```
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```



```
<ci> &gamma; </ci>
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</apply>
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```

## Derivatives

$$\frac{da}{dx} = 0$$

$$\frac{dx}{dx} = 1$$

$$\frac{d(au)}{dx} = a \frac{du}{dx}$$

$$\frac{d(u + v + w)}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx}$$

$$\frac{d(uv)}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{d(uvw)}{dx} = vw \frac{du}{dx} + uw \frac{dv}{dx} + uv \frac{dw}{dx}$$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{1}{v} \frac{du}{dx} - \frac{u}{v^2} \frac{dv}{dx}$$

$$\frac{d(u^n)}{dx} = n(u) \frac{du}{dx}$$

$$\frac{d\sqrt{u}}{dx} = \frac{1}{2\sqrt{u}} \frac{du}{dx}$$

$$\frac{d\left(\frac{1}{u}\right)}{dx} = -\frac{1}{u^2} \frac{du}{dx}$$

$$\frac{d\left(\frac{1}{u^n}\right)}{dx} = -\frac{n}{u} \frac{du}{dx}$$

$$\frac{d}{dx} = \frac{d \log(u + \sqrt{u^2 + 1})}{dx} = \frac{1}{\sqrt{u^2 + 1}} \frac{du}{dx}$$

$$\frac{d\left(\int_p^q f(x, a) dx\right)}{da}$$

## Integrals

$$\int \left(\frac{1}{x\sqrt{a^2 \pm x^2}}\right) dx = -\frac{1}{a} \log \frac{a + \sqrt{a^2 \pm x^2}}{x}$$

$$\int \left(\frac{1}{\cos(ax)(1 \pm \sin(ax))}\right) dx = \left(\frac{1}{2a(1 \pm \sin(ax))}\right) + \frac{1}{2a} \log \tan \left(\frac{\pi}{4} + \frac{ax}{2}\right)$$

## Series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

$$\forall x \in \mathbb{R} \left| e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots \right.$$

$$\forall x \in \mathbb{R} \left| (e) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots + (-1)^n \frac{x^n}{n!} \dots \right.$$

## Logs

$$\forall a > 0 \wedge b > 0 \left| \log_g ab = \log_g a + \log_g b \right.$$

$$\forall a > 0 \wedge b > 0 \left| \log_g \frac{a}{b} = \log_g a - \log_g b \right.$$

$$\forall b \in \mathbb{R} \wedge a > 0 \left| \log_g a^b = b \log_g a \right.$$

$$\forall a > 0 \quad \left| \log_g a = \frac{\log_p a}{\log_p g} \right.$$

## Goniometrics

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\sin p + \sin q = 2 \sin \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\sin p - \sin q = 2 \cos \frac{p+q}{2} \sin \frac{p-q}{2}$$

$$\cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p+q}{2} \sin \frac{p-q}{2}$$

$$2 \sin \alpha \cos \beta = \sin (\alpha + \beta) + \sin (\alpha - \beta)$$

$$2 \cos \alpha \sin \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

$$2 \cos \alpha \cos \beta = \cos (\alpha + \beta) + \cos (\alpha - \beta)$$

$$-2 \sin \alpha \cos \beta = \sin (\alpha + \beta) - \sin (\alpha - \beta)$$

$$\forall \triangle ABC \left| \frac{a}{\sin \alpha} + \frac{b}{\sin \beta} + \frac{c}{\sin \gamma} \right.$$

$$\forall \triangle ABC \left| \begin{array}{l} a^2 = b^2 + c^2 - 2bc \cos \alpha \\ b^2 = a^2 + c^2 - 2ac \cos \beta \\ c^2 = a^2 + b^2 - 2ab \cos \gamma \end{array} \right.$$